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A Review of the Attachment Line Instability for Hybrid Laminar Flow Control

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[**Abstract**] This paper focuses on the investigation of the attachment line instability for Hybrid Laminar Flow Control (HLFC), one of the most promising drag reduction technologies for modern transport aircraft respect to high Reynolds numbers and large sweep angles. The attachment line instability also plays an important role during laminar-turbulent transition control and HLFC design on a swept wing. The overview of historical research is presented and knowledge gaps are pointed out as the conclusion.

[**Keywords**] attachment line; hybrid laminar flow control (HLFC); boundary layer; subcritical instability; leading edge

Nomenclature

x, y, z	Cartesian coordinate system
u, v, w, p	Local velocities
u', v', w', p'	Disturbance velocities
W, q_∞	Free stream velocity
U, V	Mean velocities
U_e, W_e	Free-stream velocity components
X, Y, Z	Local coordinate system
Λ	Sweep angle
R	Shape semiperimeter
β	Disturbance spanwise wave number
ω	Disturbance frequency
t	Time
Re_δ	Displacement thickness Reynolds number
Re_θ	Momentum thickness Reynolds number

I Introduction

For high-subsonic speed, commercial transport aircraft, the skin friction drag represents about 50% of the total drag. Friction drag has two important generators, wings and fuselage, which have similar contributions and account for about 70% of the total friction drag (Robert, 1992).

The achievement of extensive regions of laminar flow on aircraft wings and empennages remains a long-

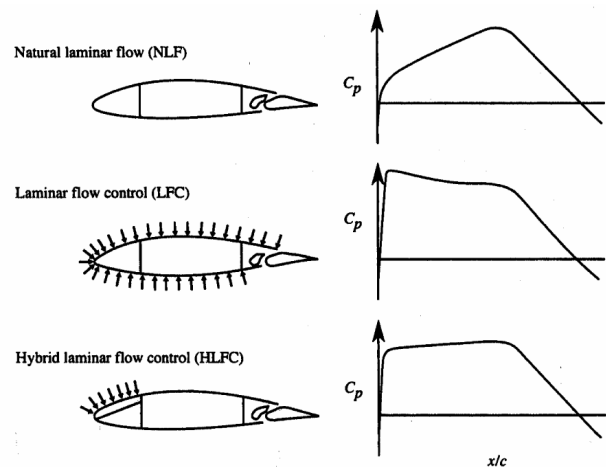


Figure 1. NLF, LFC, and HLFC concepts for wing (Joslin, 1998).

term aim as the most efficient way to reduce skin friction drag. To this end there have been considerable research and experiments in recent years into control techniques for delaying transition from laminar to turbulent flow, such as Natural Laminar Flow (NLF), Laminar Flow Control (LFC) and HLFC. The NLF employs a favorable pressure gradient (negative pressure gradient) to delay the transition process, and the LFC applies an active boundary-layer flow control (usually suction) technique to maintain the laminar state, whilst the HLFC concept basically combines suction in

the leading edge region with favourable pressure gradients after the front spar of the wing or other aircraft components like horizontal tail, fin and nacelle (Schmitt et al., 2000). Fig. 1 illustrates these concepts and shows typical pressure distributions. Generally, NLF is easier to apply but it is restricted to low sweep angle and low Reynolds numbers; LFC leads to large amount of additional systems due to the application of suction technology; it seems that with the range of Reynolds numbers and sweep angles on modern transport aircraft only the HLFC concept works out. The limits of these three technologies are demonstrated in Fig. 2.

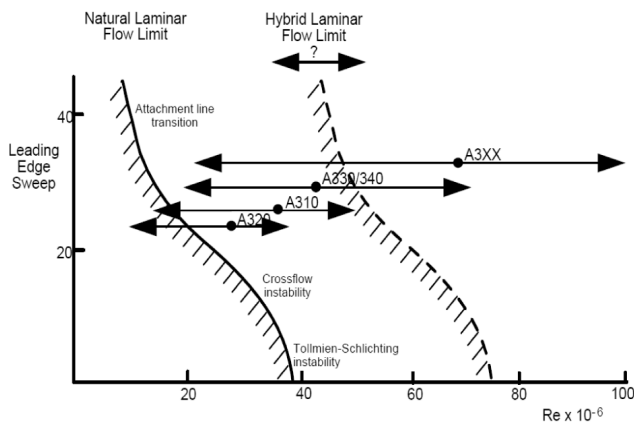


Figure 2. Limits of laminar control technologies (Wong et al., 2000).

On a swept wing, it is known that multiple instability mechanisms can operated simultaneously to cause transition from laminar to turbulent flow. The boundary-layer transition on a swept wing has been observed to occur at a location much closer to the leading edge than that on a corresponding unswept wing in flight tests (Lin et al., 1997).

If the attachment line boundary layer is turbulent, turbulence will spread over the whole wing and any attempt to control transition would be meaningless. Therefore, the attachment line instability represents a practical problem of great importance. The transition may be caused by one of three principle mechanisms which depend mainly on streamwise pressure gradient, Reynolds number and sweep angle. These three mech-

anisms are attachment line instability (or attachment line contamination), cross-flow (CF) instability and Tollmien-Schlichting (TS) instability (Wong et al., 2000), which are to be discussed more in detail in the next section.

II Background

In the boundary layer close to the leading edge, the local flow is three dimensional. The component of the velocity normal to the plane spanned by the free-stream velocity and the surface normal is called cross-flow velocity and is the cause for the complexity of the flow (Le Duc et al., 2006). The mean flow field and coordinate systems for flow past a swept-back wing is schematically shown in Fig. 3 (Sengupta et al., 2004). This section is intended to provide an overview of the constitutive features of the primary instabilities that are relevant for leading edge flow, with necessary fundamentals addressed on the attachment line topic. Following that several ways of preventing leading edge contamination are discussed especially Gaster Bump concept is depicted as an effective device in this project.

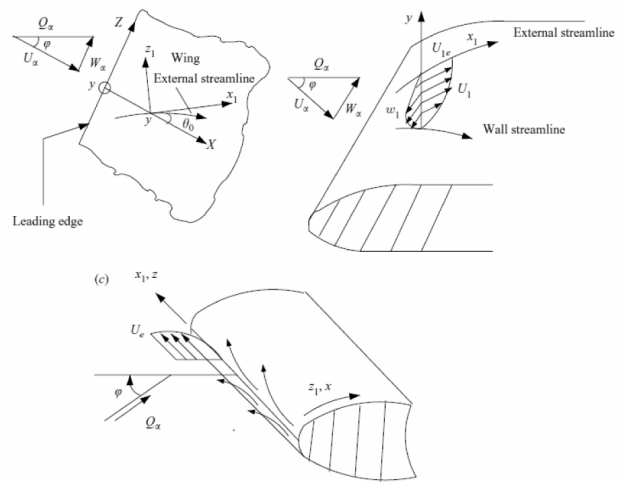


Figure 3. Flow profiles and schematic of co-ordinate systems for flow past a swept back wing (Sengupta et al., 2004). (a) Notation and coordinate system. (b) Stream wise (u) and cross-flow (w) mean velocity profiles. (c) Attachment line flow.

a. Attachment line instability. Actually it is not easy to give an accurate definition of the attachment

line. For simple geometries such as symmetrical bodies of constant chord and infinite span, it is the spanwise line along which the static pressure is maximum. An alternate definition is the line of contact at the leading edge of a swept wing where the component of flow at right angles to the leading edge is stopped. More intuitively, the attachment line represents a particular streamline which divides the flow into one branch following the upper surface and another branch following the lower surface. As a first approximation, the boundary layer flow along this line is either laminar, or turbulent, though it is not true with respect to the actual situation. For a swept wing of infinite span, assume the boundary layer properties (physical thickness, displacement or momentum thicknesses, shape factor, skin friction) are constant along the attachment line (Arnal et al., 2008b). The attachment line flow may become turbulent either through leading edge contamination or through natural instability—the latter is referred to as the attachment line instability. If transition spot were to occur on the attachment line, the outboard portion of the whole wing would be turbulent flow, that is, turbulence (or attachment-line contamination) from the fuselage boundary-layer flow can sweep out onto the attachment line and cause the entire wing to be turned into turbulent flow (Joslin, 1998).

The attachment line instability is a viscous, linear instability. Hall, Malik, and Poll (1984) were the first obtaining numerical solutions by linearizing the Navier-Stokes equations upon substitution of GH structure and using an eigenvalue-problem approach. The base flow called swept Hiemenz flow. This is a stagnation point flow with a superimposed spanwise component and an exact solution to the incompressible Navier-Stokes equations, hence, its use is advantageous in stability analyses, see Fig. 4 (Le Duc et al., 2006).

Consider the attachment line boundary layer at large distances from its origin, based on Hiemenz flow. Z is the spanwise direction, X is the direction normal to it, $X = 0$ corresponding to the attachment line. U and W designate the projections of the mean velocity along X and Z , respectively. Along the attachment

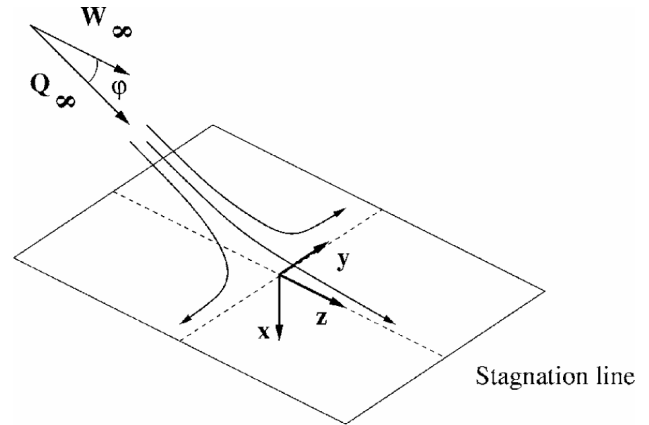


Figure 4. Sketch of the swept Hiemenz flow (Duc et al., 2006). The flow impinges with a speed Q_∞ and sweep angle φ . W_∞ is the velocity component along the span. z runs along the span, x is normal to the plate and y indicates the chordwise direction.

line, W represents the streamwise velocity, whilst U is the crossflow component. The free-stream velocity components U_e and W_e around the attachment line are given by $U_e = kX$ and $W_e = W = \text{constant}$. The simplest idea is to introduce small disturbances as the Tollmien-Schlichting (TS) waves. In the framework of the spatial theory:

$$(u', v', w', p') = (u(y), v(y), w(y), p(y)) \exp(\sigma Z) \exp[i(\beta Z - \omega t)] \quad (1)$$

This relationship represents periodic fluctuations of wavenumber β and frequency ω , growing ($\sigma > 0$) or decaying ($\sigma < 0$) in the spanwise direction Z . Introducing this expression in the Navier-Stokes equations, linearizing in u, v, w and using the parallel flow approximation lead to the classical fourth-order Orr-Sommerfeld equation written for the attachment line mean velocity profile W/W_e (Arnal et al., 2008b).

The vertical mean velocity component V and the X and Z -derivatives of the basic flow are neglected in the parallel flow approximation. But these assumptions are not correct for the attachment line flow. The mean flow of the attachment line boundary layer is uniform in the spanwise direction Z , whilst non-zero vertical velocity component exists, and U at a fixed altitude, y is linear respect to X . It is possible to follow a more rigorous approach by considering a special class of small dis-

turbances. The first mode is introduced by Görtler (1955) and Hämmerlin (1955), also known as GH temporal model. These Görtler-Hämmerlin (GH) disturbances are of the form:

$$\begin{aligned} u' &= Xu(y) \exp(\sigma Z) \exp[i(\beta Z - \omega t)] \\ (v', w', p') &= (v(y), w(y), \\ &p(y)) \exp(\sigma Z) \exp[i(\beta Z - \omega t)] \quad (2) \end{aligned}$$

Introducing (2) into the Navier-Stokes equations and linearizing the sets, lead to another eigenvalue problem, but, by contrast with the Orr-Sommerfeld approach, the system of ordinary differential equations is obtained without using the parallel flow approximation. In other words, the GH disturbances are exact solutions of the linearized Navier-Stokes equations. This system is of sixth-order, while the Orr-Sommerfeld equation is fourth-order (Arnal et al., 2008b).

The principal result of a linear stability analysis is a critical Reynolds number, above which the flow is unstable against all disturbances, however infinitesimal. Information about wavelengths and growth rates of disturbance is found, whilst unfortunately the important information such as amplitudes of the disturbances, its influence on the base flow and the nonlinear interaction of the modes is destroyed by the linearization (Le Duc et al., 2006).

To evaluate the critical Reynolds number, there are different definitions to be clarified:

Momentum Thickness Reynolds Number:

$$Re_\theta = \frac{\rho u_\infty \delta}{\mu}; \quad \delta_1 = \int_0^\infty \frac{\rho u}{\rho_m u_\infty} \left(1 - \frac{u}{u_\infty}\right) dy \quad (3)$$

Displacement Thickness Reynolds Number:

$$Re_\delta = \frac{\rho u_\infty \delta}{\mu}; \quad \delta_2 = \int_0^\infty \left(1 - \frac{u\rho}{u_\infty \rho_\infty}\right) dy \quad (4)$$

It is common control practice to specify the Reynolds number:

$$Re_\theta = 0.404 \sqrt{\frac{q_\infty R \sin^2 \Lambda}{(1 + \varepsilon) \cos \Lambda}} \quad (5)$$

which is based on the approximate momentum thickness less than 100 (or a Reynolds number based on displacement thickness, $Re_\delta = 245$). In this equation, ε denotes the eccentricity of an ellipse with semiperimeter, R fitted into the shape of the leading edge, is the

sweep angle, and q_∞ is the freestream velocity (Le Duc et al., 2006). In summary, based on the theoretical and experimental studies, the critical number for the two-dimensional linear instability of subsonic flows is deemed $Re_\delta = 245$ (Joslin, 1998).

b. Cross-flow instability. Many theoretical, numerical and experimental investigations have demonstrated the transition is triggered by the breakdown of unstable waves generated by the disturbances. Cross-flow (CF) instability (also the CF waves) are unstable in regions of negative pressure gradient (accelerating), typically in the vicinity of the leading edge. It is an inviscid, linear instability which is due to the inflection point in the cross-flow velocity profile, see Fig. 5. Its base flow is a three-dimensional (3D) boundary layer. These instabilities are strongly dependent on wing leading edge sweep and the initial flow acceleration corresponding to a steep favourable pressure gradient. The present practice to control the cross-flow instability is to modify the mean flow profile (Lin et al., 1997, Duc et al., 2006, Wong et al., 2000). Ideally, the transition location may be postponed downstream by distributing surface imperfection, so that preferable slower modes are growing and the most unstable ones are decreasing.

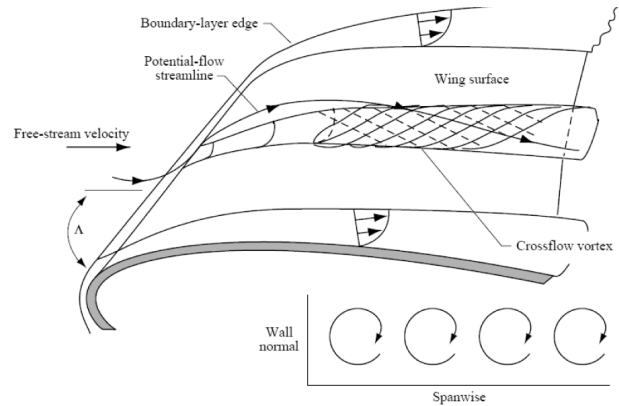


Figure 5. Sketch of cross-flow vortices over swept wing (Joslin, 1998). The flow impinges with a speed Q_∞ and sweep angle φ . W_∞ is the velocity component along the span. z runs along the span, x is normal to the plate and y indicates the chordwise direction.

Both the cross-flow and the attachment line insta-

bility can exist for one eigenvalue problem, which indicates a relationship of the two. As their structures are similar, the different performances may depend on whether the viscous or the dynamic nature (inflection point) of the instability dominates (Bertolotti, 1999, Le Duc et al., 2006).

c. Tollmien-Schlichting instability. TS instability (also the TS waves) are the result of the instability of the boundary layer streamwise mean velocity profiles; they develop in regions of zero or positive pressure gradients. If the CF instability has been controlled and the onset of transition delayed to aft of the minimum pressure point, then TS instability would become the dominant mode for transition. As favorable pressure gradient leads to decreased TS-disturbance growth and increased CF-disturbance growth (Arnal, 1992), NLF wing design engineer would seek to optimize the mean flow profile to get the pressure distribution and sweep for prescribed Reynolds number and Mach number such that the pressure gradient causes the minimum growth of both the TS and CF disturbances.

In summary, for wing sweeps from $0 \sim 10$ degree, TS disturbance amplify and cause natural transition. Between wing sweep $10 \sim 30$ degree, both TS and CF disturbances are present, amplify, and cause transition; much of the nonlinear interact mechanisms are unknown. For wings swept greater than 30 degree, CF disturbance dominate, often cause transition near the leading edge of the wing (Joslin, 1998).

d. Streamline curvature instability. At sufficiently large streamline curvature a further instability in the vicinity of the stagnation line is observed to give the lowest values of the critical Reynolds number except for a very narrow region close to the attachment line, where the viscous and cross-flow instabilities are dominant (Itoh, 2006). The mechanism is similar to the one of the cross-flow instability and hard to distinguish the two. However, it is not clear how this instability relates other instabilities in various 3D flows.

III Historical Research

Early studies have focused on experimental ef-

forts. Pfenninger (1965) found the lamimar flow could be obtained below $Re_\theta \approx 100$ (or a Reynolds number based on displacement thickness, $Re_\delta = 245$) while above the leading edge contamination occurred. Gaster (1967) examined the small amplitude disturbance problem by using acoustic excitation along the attachment line of a swept cylinder model. He concluded that the small-amplitude disturbances in an attachment line boundary layer were stable for momentum-thickness Reynolds numbers Re_θ below 170. Later, Cumpsty and Head (1969) experimentally studied large amplitude disturbances, and observed that laminar flow was stable to small-amplitude disturbances up to $Re_\delta \approx 245$. In the same year, Pfenninger and Bacon (1969) used a wing swept to 45 degree, and reported that the transition occurred at about $Re_\delta \approx 240$. A continued study on the transition initiated near the attachment line of swept wings by Poll and Paisley (1985) showed no unstable modes were observed below $Re_\delta \approx 230$. Similar critical Reynolds numbers were reported by Arnal et al. (1992) with a swept wing model and Maddalon et al. (1989) with the Jestar LFC flight test aircraft.

While the critical Reynolds number was investigated by experimental researchers, a viscous instability mechanism clearly distinct from the cross-flow instability was observed (Pfenninger et al., 1969, Poll et al., 1985, Bippes, 1990). In this case, a part of the leading edge is laminar, another part is transitional (intermittent) and a third part is turbulent. As these mechanisms present strong similarities with those observed in a flat plate boundary layer with a low free-stream turbulence level, that means it is possible to develop a linearized stability theory.

This study was followed by theoretical investigations into the stability of flow in the vicinity of the attachment line. Poll (1979) adopts a parallel flow assumption, using the spanwise and normal velocity profile. Both chordwise (U) and wall-normal (V) velocity components of the basic flow were neglected so that the stability equations simply reduced to the classical Orr-Sommerfeld equation. The critical Reynolds number Re_δ was predicted to be close to 270 based on mo-

mentum thickness along the attachment line. Hall, Mlik, and Poll (1984) who have determined a semi-analytical Navier-Stokes solution called swept Hiemenz flow. They solved the linear stability equations obtained by assuming that the chordwise component of the perturbation velocity u' depends linearly on the chordwise coordinate x . TS wave is introduced as in Eqn. 6.

$$\begin{aligned} (u', v', w') = & (u(y), v(y), \\ & w(y)) \exp[i(\beta Z - ct)] \end{aligned} \quad (6)$$

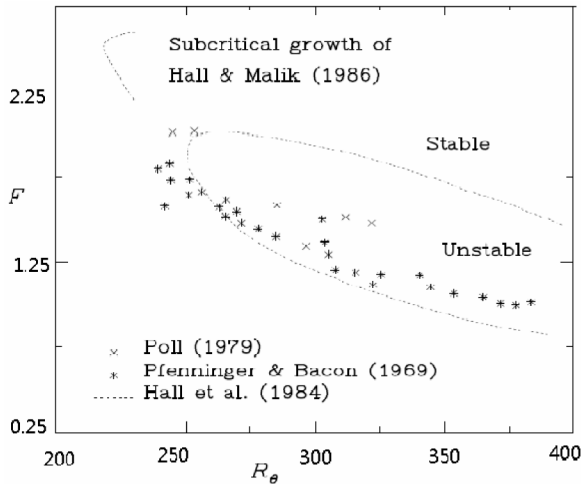
where β is the spanwise wavenumber of the disturbance. They proposed a critical $Re_\delta = 583.1$ for the appearance of a modal temporal instability under the Görtler-Hämmerlin assumption. Theofilis (1993) first assessed the feasibility of a direct numerical simulation in studying the stability of leading edge boundary layer flow, then he (1995) computed perturbations around the incompressible Hiemenz flow and adopted a similarity model for the perturbations in the x -direction, such that the perturbation eigensolution could be computed using a one-dimensional mathematical model. Sparlart (1988) was the first who performed temporal nonlinear three dimensional direct numerical simulations (DNS) for the incompressible attachment line boundary layer flow. By trying a number of simulations near the critical Reynolds number, he obtained results consistent with linear stability prediction based on the similarity model (Heeg, 1998). In further study of Sparlart (1990), he studied crossflow instability by using different types of disturbances which generated streamwise vortices. Joslin (1995) reported similar results in his three-dimensional computations within the spatial framework. Lin and Malik (1996) extended to compute more general flow perturbations but no further modal instabilities have been found beyond the Görtler-Hämmer type. The results indicated that unstable perturbations other than the special symmetric two-dimensional mode referred to results got by Hall et al. (1984) do exist in the attachment line boundary layer provided $Re_\delta > 646$, nevertheless the explanation why additional modes exist wasn't given. They also pointed that both symmetric and antisymmetric two—and three-dimensional eigenmodes can be amplified, whilst the

symmetric two-dimensional mode always has the highest growth rate and dictates the instability. The existence of one of these modes was subsequently confirmed in a direct numerical simulation by Joslin (1996). Lin and Malik (1997) extended their work by investigating the effect of leading-edge curvature. They used the second order boundary layer theory to account for the curvature effects on the mean flow and applied a two-dimensional eigenvalue approach to solve the linear stability equations which is fully account for the effects of non parallelism and leading-edge curvature. Finally they concluded that the leading-edge curvature has a stabilizing influence on the attachment line boundary layer. By either using a smaller leading edge radius or by increasing sweep the Reynolds number based on leading edge radius decreases, consequently the critical Reynolds number increases.

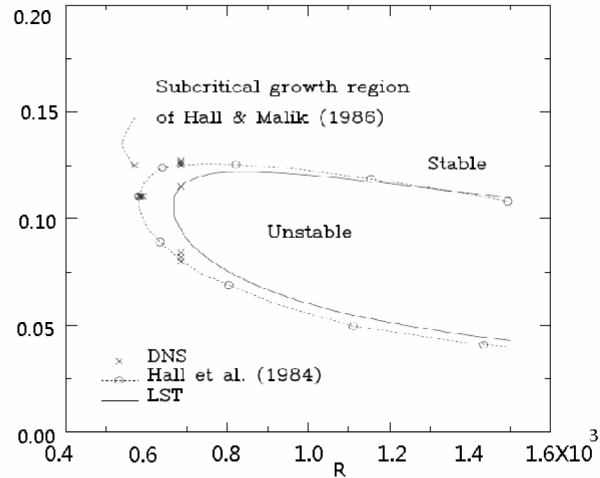
Apparently, there is a significant discrepancy of the critical Reynolds number between experimental value and linear theoretical value. It indicates the actual transition is a subcritical mechanism. Hall and Malik (1986) have presented analysis and two-dimensional computations, and the results showed the flow of $Re_\delta \in [535, 583]$ could be subcritically destabilized. The region of subcritical instability growth is shown in Fig. 6 (a), with the experiments of Pfenninger and Bacon (1969) and Poll (1979, 1980) and the neutral curve of Hall et al. (1984). Consistent with the Pfenninger and Bacon (1969) experiments, large amplitude disturbances became unstable before the linear critical point (subcritical). Furthermore, near the lower branch of the neutral curve, Hall and Malik (1986) observed equilibrium states for large amplitude disturbances. While nonlinear subcritical two-dimensional equilibria were confirmed by Joslin (1996a) in his DNS (Fig. 6 (b)) and by Balakumer (1998) in his secondary instability analysis, the other two-dimensional DNS carried by Jiménez (1990) and Theofilis (1998) under the GH assumption failed to find such solution. Consequently, largely academic interest was drawn onto two counts. First, the role of three-dimensionality in the swept attachment line boundary layer

needs to be clarified, which was demonstrated in the work of Hall and Seddougui (1990) under high Reynolds number approximation but remained unknown at finite Reynolds numbers. Second, two-dimensional nonlinear equilibria were reported as ceasing to exist below $Re_\delta \approx 535$ by Hall and Malik (1986), and $Re_\delta \approx 511$ by Balakumar (1998), leaving $Re_\delta \in [245, 511]$ unexplained. Thus further study needs to be done to fill this gap or that with linear theory of Re_δ values

between 245 and 583 (Theofilis et al., 2003). To this end, Joslin (1996b) introduced a ‘bypass’ transition to describe the interactions of multiple three-dimensional modes. On the other hand, Theofilis et al. (2003) proposed an extended Görtler-Hämmer model which aimed to recover the three-dimensional linear instability characteristics. They also demonstrated the polynomial structure of individual three-dimensional extended GH eigenmodes by DNS under linear conditions.



a) Neutral curve, experimental regions of instability growth, and theoretical region of subcritical growth in attachment line boundary layer



b) Neutral curves, region of subcritical instability growth, and sample points for DNS in attachment line boundary

Figure 6. The region of subcritical instability growth (Joslin, 1996a).

Besides the interest on Reynolds number, more recently, the potential for temporal amplification of disturbance energy in the subcritical regime has been investigated by Obrist and Schmid (2003) using a non-modal stability analysis and by Guégan et al. (2006, 2007) using a variational formulation based on the direct and adjoint linearized Navier-Stokes equations, providing the evidence that short-term temporal instability mechanisms may play an important role in the transition process. Based on these studies, Guégan et al. (2008) studied the spatial non-modal stability problem. They claimed a lift-up mechanism which favours steady perturbations with a chordwise scale that quantitatively matches its counterpart for classical Blasius boundary layers.

Moreover, so far not much is known about the effect of compressibility on the attachment line instabil-

ity as well as the exact mechanism of the instability modes. Le Duc et al. (2002) experimented the weak compressible regime on a flat plate, confirming a critical Reynolds number of 644. Sesterhenn and Friedrich (2006) reported the typical Goertler-Haemmerlin modes are recovered at the attachment line and found to be stabilised with an increase of the Mach number and reduction of the nose radius. Mack et al. (2008) presented results from global stability analysis of compressible flow around a swept parabolic body, and demonstrated the connecting attachment line and cross-flow modes which is known little so far. A more detailed information refers to the work of Le Duc et al. (2006).

Meanwhile, many devices are constructed and tested to prevent leading edge contamination, among which the Gaster bump proposed by Gaster (named ac-

cordingly) (Gaster, 1965) is deemed as a successful device. This consists of a small fairing at the wing-fuselage junction, shaped in a way that a stagnation point on the leading edge near the root is created, where the disturbances coming from the fuselage are deviated and a new laminar boundary layer is generated. A bump designed by ONERA (Office National d'Etudes et de Recherches Aérospatiales) around 1990 is illustrated in Fig. 7. It had reached a critical Reynolds number around 460 (Arnal et al., 2008a).

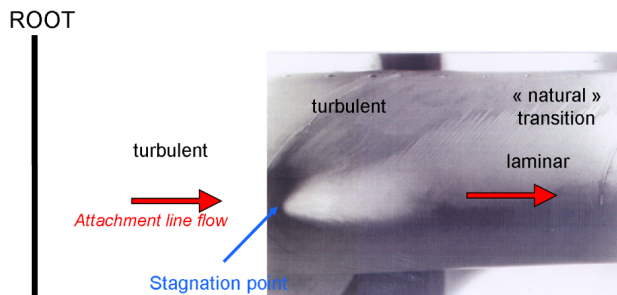


Figure 7. Gaster Bump on the leading edge of a swept wing (Arnal et al., 2008a).

Other passive anti-contamination devices (Gaster, 1998) and active devices exploiting suction have been evaluated in various ways, more information can be found in work of Arnal et al. (2008).

IV Conclusion

Theofilis et al. (2003) has concluded that ‘there exist two most significant unresolved issues in the stability of the attachment line boundary layer’. The first relates to the subcritical instability, and the second is the relationship of the instability at attachment line to the downstream events in the chordwise direction. Both of them appear to be beyond the classical one-dimensional analysis based on GH structure.

The following topics are reported as the faced knowledge gap before engineering application of HLFC:

1) The explanation of nonlinear subcritical two-dimensional equilibria where conflict exists in the DNS studies, and the role of three-dimensionality in the

swept attachment line boundary layer;

2) The explanation of the subcritical instability;

3) The mechanisms of streamline curvature instability and other primary instability modes and their interactions;

4) The effects of compressibility on the attachment line boundary layer;

5) The most efficient way to avoid leading edge contamination and natural transition;

The HLFC concept has been evaluated in large amount of wind tunnel tests and flight tests, however, it is the first application of this technology on a large commercial transport aircraft quite recently when it is introduced on the stretched 787-9, with a passive suction system. This indicates that there is still a long way for HLFC technology from the laboratory to the aeronautical application. Unfortunately, the theory imperfection of the flow instability makes this way even more challenging. Due to the great importance of the attachment line boundary layer, more work should be done in the instability field, including the exact flow mechanisms and extension or innovation of the instability theory.

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